TYING IT ALL TOGETHER: REFRESHER FOR UNIT 5 KEY

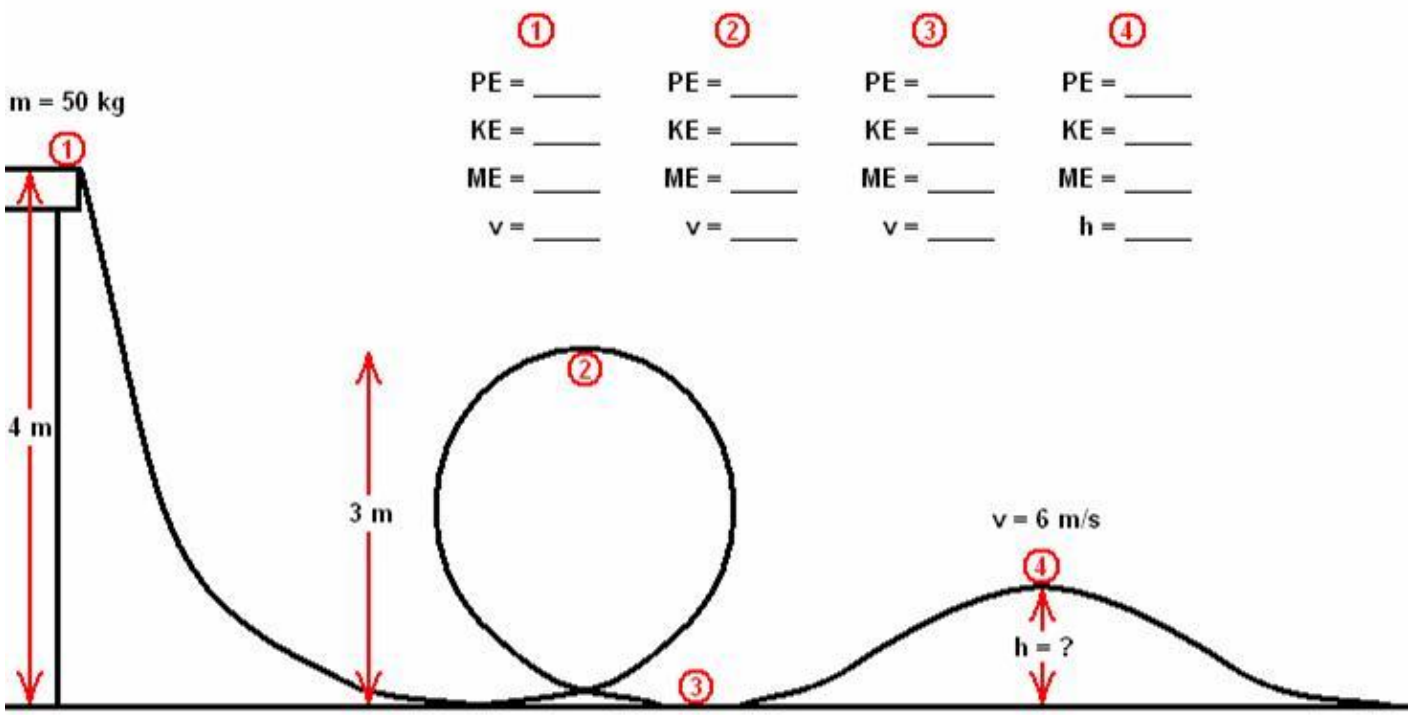
***Instructions**: Show all of your work completely in your journal, including the equations used in variable form. Pay attention to sig figs and units; use complete sentences if applicable.*

1. You are rolling bowling balls toward each other for fun (like physics teachers do in their spare time) at Lucky Strike Lanes. You take an 8.0 kg ball and roll it at 2.0 $^{m}/_{s}$ toward a 12 kg bowling ball at rest. If the 12 kg ball has a final velocity of 1.5 $^{m}/_{s}$, calculate the velocity of the 8.0 kg ball. What type of collision is this?

   This is an elastic collision:

$$p_{before} = p_{after} \rightsquigarrow m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$
$$(8.0\text{ kg})(2.0\ ^{m}/_{s}) + 0 = (8.0\text{ kg})v'_1 + (12\text{ kg})(1.5\ ^{m}/_{s})$$
$$(16\text{ kg}\,^{m}/_{s}) - (18\text{ kg}\,^{m}/_{s}) = -2.0\text{ kg}\,^{m}/_{s} = (8.0\text{ kg})v'_1$$
$$\boxed{\boldsymbol{v'_1 = -0.25\ ^{m}/_{s}}}$$

2. Use conservation of energy to fill in the blanks for the diagram below. Show all of your work!



Point 1:

$$E_P = mgh = (50\text{ kg})\left(9.80\ ^{m}/_{s^2}\right)(4\text{ m}) = \boxed{1960\text{ J}}$$
$$E_K = \frac{1}{2}mv^2 = \boxed{0\text{ J}} \rightsquigarrow \boxed{v = 0\ ^{m}/_{s}}$$
$$ME = E_K + E_P = \boxed{1960\text{ J}}$$

Point 2:

$$\boxed{ME = constant = 1960\text{ J}}$$

$$E_P = mgh = (50\text{ kg})\left(9.80\ {}^{\text{m}}/_{\text{s}^2}\right)(3\text{ m}) = \boxed{1470\text{ J}}$$

$$ME = E_K + E_P \rightsquigarrow E_K = ME - E_P = 1960\text{ J} - 1470\text{ J} = \boxed{490\text{ J}} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2(490\text{ J})}{(50\text{ kg})}} = \boxed{4.4\ {}^{\text{m}}/_{\text{s}}}$$

Point 3:

$$\boxed{ME = constant = 1960\text{ J}}$$

$$E_P = mgh = (50\text{ kg})\left(9.80\ {}^{\text{m}}/_{\text{s}^2}\right)(0\text{ m}) = \boxed{0\text{ J}}$$

$$\boxed{E_K = 1960\text{ J}} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2(1960\text{ J})}{(50\text{ kg})}} = \boxed{8.9\ {}^{\text{m}}/_{\text{s}}}$$

Point 4:

$$\boxed{ME = constant = 1960\text{ J}}$$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}(50\text{ kg})(6\ {}^{\text{m}}/_{\text{s}})^2 = \boxed{900\text{ J}}$$

$$ME = E_K + E_P \rightsquigarrow E_P = ME - E_K = 1960\text{ J} - 900\text{ J} = \boxed{1060\text{ J}} = mgh$$

$$h = \frac{E_P}{mg} = \frac{1060\text{ J}}{(50\text{ kg})\left(9.80\ {}^{\text{m}}/_{\text{s}^2}\right)} = \boxed{2.2\text{ m}}$$

3. In ballistics labs, the muzzle velocity of guns (the velocity of the bullet right as it leaves the gun) is often found by firing the bullet into a massive block of wood on a frictionless surface and measuring the final velocity of the block.
   a. What type of collision is this?

      Inelastic collision

   b. Given that the mass of the bullet is 13 grams, the mass of the block is 4.0 kg and the final velocity of the block is 1.2 ${}^{\text{m}}/_{\text{s}}$, find the initial velocity of the bullet.

$$p_{before} = p_{after} \rightsquigarrow m_1v_1 + m_2v_2 = (m_1 + m_2)\,v'$$

$$(0.013\text{ kg})v_1 + (4.0\text{ kg})(0\ {}^{\text{m}}/_{\text{s}}) = (0.013\text{ kg} + 4.0\text{ kg})(1.2\ {}^{\text{m}}/_{\text{s}})$$

$$(0.013\text{ kg})v_1 = 4.8\text{ kg}\,{}^{\text{m}}/_{\text{s}}$$

$$\boxed{\boldsymbol{v_1 = 370\ {}^{\text{m}}/_{\text{s}}}}$$

   c. You know that the block brought the bullet to rest by doing work on the bullet. Calculate the work done on the bullet by the block.

$$W = \Delta E_K = \frac{1}{2}m({v_2}^2 - {v_1}^2)$$

$$W = \frac{1}{2}(0.013\text{ kg})(0 - (370\ {}^{\text{m}}/_{\text{s}})^2)$$

$$\boxed{\boldsymbol{W = 890\text{ N}\cdot\text{m}}}$$

d. If the bullet penetrated the block 20.0 cm, what force did the block exert on the bullet?

$$W = F \cdot d \rightsquigarrow F = \frac{W}{d} = \frac{890\ \text{N} \cdot \text{m}}{0.200\ \text{m}}$$

$$\boxed{\boldsymbol{F = 4500\ \text{N}}}$$

e. Describe how momentum and energy are conserved in this situation.

Kinetic energy is not conserved (since it's an inelastic collision). Mechanical energy is converted into another form (most likely sound and heat) so energy is conserved overall. Momentum of the system overall is conserved; we must consider the momentum of the bullet AND the block of wood.

4. A 1200 kg car is crash-tested against a rigid wall. The car is accelerated by a cable underneath it, which provides a constant force of 500. N for a distance of 15.0 m.

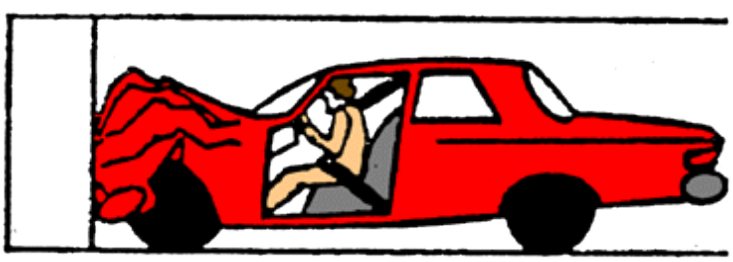
a. What is the velocity just before it hits the wall?

$$W = \Delta E_K \rightsquigarrow F \cdot d = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$v = \sqrt{\frac{2(F \cdot d)}{m}} = \sqrt{\frac{2(500\ \text{N})(15.0\ \text{m})}{1200\ \text{kg}}}$$

$$\boxed{\boldsymbol{v = 3.54\ {}^{\text{m}}/_{\text{s}}}}$$

b. The car's "crumple zone" crumples 2.30 m upon impact. What is the force the car experiences upon impact?



$$W = \Delta E_K \rightsquigarrow F \cdot d = \frac{1}{2} m({v_2}^2 - {v_1}^2)$$

$$F = \frac{\frac{1}{2} m({v_2}^2 - {v_1}^2)}{d} = \frac{\frac{1}{2}(1200\ \text{kg})(0^2 - (3.54\ {}^{\text{m}}/_{\text{s}})^2)}{(2.30\ \text{m})}$$

$$\boxed{\boldsymbol{F = -3210\ \text{N}}}$$

5. Pat is ready for spring training! The ball is pitched at 45 $^{\mathrm{m}}/_{\mathrm{s}}$ and he swings his bat with an initial speed of 31 $^{\mathrm{m}}/_{\mathrm{s}}$. After the bat and the ball collide, the ball leaves the bat at homerun velocity, 67 $^{\mathrm{m}}/_{\mathrm{s}}$. The time of contact is 0.0015 sec. The mass of the bat is 1.0 kg and the mass of the ball is 0.14 kg.
   a. What is the change in momentum of the baseball?

$$\Delta p = m \cdot \Delta v = m(v_2 - v_1) = (0.14\ \mathrm{kg})(67\ {}^{\mathrm{m}}/_{\mathrm{s}} - -45\ {}^{\mathrm{m}}/_{\mathrm{s}})$$
$$\boxed{\boldsymbol{\Delta p = 15.7\ \mathrm{N \cdot s}}}$$

   b. What is the force of impact of the bat against the ball?

$$\boxed{\boldsymbol{I = \Delta p = 15.7\ \mathrm{N \cdot s}}}$$

   c. By how much is the bat slowed down by the impact?

$$I = \Delta p = m \cdot \Delta v \rightsquigarrow \Delta v = \frac{I}{m}$$
$$\Delta v = \frac{15.7\ \mathrm{N \cdot s}}{1.0\ \mathrm{kg}}$$
$$\boxed{\boldsymbol{\Delta v = 15.7\ {}^{\mathrm{m}}/_{\mathrm{s}}}}$$

6. Patty is looking to play a trick on Pat by dropping a water balloon on his head. Her plan is to climb a tree, sit on a branch and drop the water balloon as Pat walks underneath. Sounds good, huh? ☺
   a. If she carries this 0.75 kg balloon up a tree 15 m vertically, how much work has she done to the balloon?

$$W = \Delta E_P = mg(h_2 - h_1) = (0.75\ \mathrm{kg})\left(9.80\ {}^{\mathrm{m}}/_{\mathrm{s}^2}\right)(15\ \mathrm{m} - 0\ \mathrm{m})$$
$$\boxed{\boldsymbol{W = 110\ \mathrm{N \cdot m}}}$$

   b. When Patty drops the balloon on Pat's head (approximately 2.0 m above the ground), how fast will the balloon be traveling? *(Hint: Use energy equations!)*

$$E_{K1} + E_{P1} = E'_{K2} + E'_{P2}$$
$$0 + 110\ \mathrm{J} = \frac{1}{2}m{v_2}^2 + mgh_2 = \frac{1}{2}(0.75\ \mathrm{kg}){v_2}^2 + (0.75\ \mathrm{kg})\left(9.80\ {}^{\mathrm{m}}/_{\mathrm{s}^2}\right)(2.0\ \mathrm{m})$$
$$\boxed{\boldsymbol{v_2 = 16\ {}^{\mathrm{m}}/_{\mathrm{s}}}}$$

   c. If Pat thinks quick, dodges and catches the balloon with a downward motion of his hands, such that he exerts a constant force on the balloon for 0.30 seconds, what is the magnitude of this force? *(Hint: think impulse!)*

$$I = \Delta p \rightsquigarrow F \cdot \Delta t = m(v_2 - v_1)$$
$$F = \frac{m(v_2 - v_1)}{\Delta t} = \frac{(0.75\ \mathrm{kg})(0 - 16\ {}^{\mathrm{m}}/_{\mathrm{s}})}{(0.30\ \mathrm{s})}$$
$$\boxed{\boldsymbol{F = -40\ \mathrm{N}}}$$

d. Why would the balloon break if it hit Pat's head, but probably not if he caught it with a downward motion? Use appropriate physics terminology in your answer.

By catching it with a downward motion, he is increasing the time of contact. Our impulse equation shows us that this will decrease the force, thus making it less likely the balloon will break.