Warm up 03.30.2018

A 25 g ball is fired with an initial speed v_1 toward a 150 g ball that is hanging motionless from a 1.50 m string.

The balls have a perfectly elastic collision. As a result, the 150 g ball swings out until the string makes an angle of 37° with the vertical.

What was v_1?

Warm up 03.30.2018

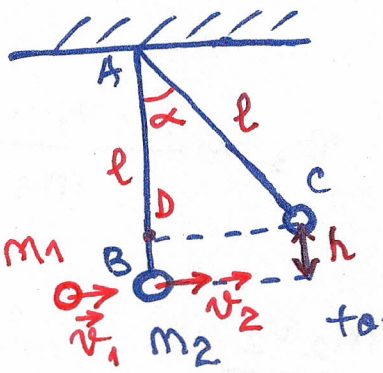
$m_1 = 25\,g$

$m_2 = 150\,g$

$\alpha = 37^\circ$

$\ell = 1.50\,m$

$v_1 = ?$



Momentum is conserved in all collisions.

CONSERVATION OF MOMENTUM

$$\underbrace{m_1 v_1}_{\text{total before collision}} = \underbrace{m_1 v_3 + m_2 v_2}_{\text{total after collision}} \quad (1)$$

The second ball will move from point B to point C until all kinetic energy from point B will be converted in potential energy in point C. (conservation of energy)

$$E_B = E_C$$

$E_B = \frac{m_2 v_2^2}{2}$ (no potential energy)

$E_C = m_2 g h$ (no kinetic energy)

$$\Rightarrow \frac{m_2 v_2^2}{2} = m_2 g h$$

$$\boxed{v_2 = \sqrt{2gh}}$$

$h = AB - AD$

$AB = \ell$

$AD = \ell \cdot \cos\alpha$

$$\Rightarrow h = \ell - \ell\cos\alpha = \ell(1 - \cos 37^\circ)$$

$$h = 1.50\,m\,(1 - \cos 37^\circ) = 1.50\,m\,(1 - 0.8) = 0.3\,m$$

$$v_2 = \sqrt{2 \cdot 9.8 \cdot 0.3}\ m/s = 2.42\ m/s$$

$$\boxed{v_2 = 2.42\ m/s}$$

During the elastic collision between ball 1 and 2, the energy is conserved

$$\frac{m_1 \cdot v_1^2}{2} = \frac{m_1 v_3^2}{2} + \frac{m_2 \cdot v_2^2}{2} \quad (2)$$

Extracting v_3 from equation (1): $v_3 = \frac{m_1 v_1 - m_2 v_2}{m_1}$

Replacing v_3 in equation (2)

$$\frac{m_1 v_1^2}{2} = \frac{m_1}{2}\left(\frac{m_1 v_1 - m_2 v_2}{m_1}\right)^2 + \frac{m_2 v_2^2}{2}$$

$$\frac{m_1 v_1^2}{2} = \frac{m_1}{2}\left(\frac{m_1^2 v_1^2 + m_2^2 v_2^2 - 2 m_1 m_2 v_1 v_2}{m_1^2}\right) + \frac{m_2 v_2^2}{2} \quad \Big| \times 2 m_1$$

$$m_1 v_1^2 = m_1^2 v_1^2 + m_2^2 v_2^2 - 2 m_1 m_2 v_1 v_2 + m_2 v_2^2 \cdot m_1$$

$$2 m_1 m_2 v_1 v_2 = m_2^2 v_2^2 + m_2 m_1 v_2^2$$

$$v_1 = \frac{m_2 v_2^2 (m_2 + m_1)}{2 m_1 m_2 v_2} = \frac{v_2 (m_2 + m_1)}{2 m_1}$$

$$v_1 = \frac{v_2 (m_2 + m_1)}{2 m_1}$$

$$v_1 = \frac{2.420475 \text{ kg}}{2 \cdot 0.025 \text{ kg}} \text{ m/s} = \frac{0.4235}{0.05} \text{ m/s} = 8.47 \text{ m/s}$$

$$v_1 = 8.47 \text{ m/s}$$